Intelligent ELECTRE III Multi-Attribute Decision Making with Neural Network Threshold Detection and Multiprocessing Parallel Computation

Meng-xian Wang[1,*], Jian-qiang Wang[1], Xiao-hui Wu[1], Lin Li[2]

[1] School of Business, Central South University, Changsha 410083, Hunan, China

[2] School of Management, Hunan University, Changsha 410082, Hunan, China

[*] Corresponding author: mxwang@csu.edu.cn

Abstract

Multi-attribute decision-making (MADM) methods are fundamental tools for complex industrial and organizational decision problems, yet classical approaches face critical limitations when confronted with the scale and complexity demands of contemporary AI-driven environments. The ELECTRE III method, renowned for its nuanced treatment of preference heterogeneity through indifference, preference, and veto thresholds, suffers particularly from two structural challenges: the O(n^2) computational complexity of pairwise comparison that becomes prohibitive for large alternative sets, and the persistent difficulty of determining appropriate threshold parameter values for real-world applications. This paper proposes an intelligent ELECTRE III framework that addresses both limitations through integrated neural network and multiprocessing innovations. A multi-layer perceptron neural network is trained to automatically detect the three ELECTRE III threshold parameters (q, p, v) from criteria weight distributions and historical decision data, eliminating the expert elicitation bottleneck. A multiprocessing-based parallel ELECTRE III engine partitions the pairwise concordance and discordance matrix computations across available CPU cores, achieving near-linear speedup scaling. Evaluation on the QS World University Rankings dataset (n = 500 universities, 6 criteria) demonstrates that the neural network threshold detector achieves mean absolute errors of 0.011--0.021 across the three threshold types, while 8-core multiprocessing achieves 6.5× speedup at 1,000 alternatives. Comparative analysis against TOPSIS, VIKOR, PROMETHEE, AHP, and SAW confirms ranking consistency (Spearman rho > 0.88) while uniquely preserving the veto and incomparability structures that distinguish ELECTRE III from compensatory MADM methods. The proposed framework provides a practical pathway for deploying intelligent ELECTRE III in large-scale industrial supplier evaluation, technology selection, and strategic investment prioritization contexts.

Keywords: multi-attribute decision-making; ELECTRE III; neural network; threshold detection; multiprocessing; intelligent decision support; supplier evaluation

1. Introduction

The proliferation of data-intensive decision scenarios in modern industrial operations has created an urgent demand for decision support methodologies capable of operating at scale without sacrificing the nuanced preference modeling that distinguishes high-quality decisions from simple optimization [1,2]. Supplier qualification in global supply chains, equipment technology selection for digital manufacturing upgrades, R&D portfolio prioritization, and strategic investment allocation are representative examples of industrial MADM problems that may involve hundreds or thousands of alternatives evaluated against multiple competing criteria with non-linear preference structures [3,4].

The ELECTRE (Elimination and Choice Translating Reality) family of outranking methods, developed by Roy and colleagues, occupies a distinctive position in the MADM landscape through its non-compensatory preference model and explicit treatment of incomparability--the recognition that some pairs of alternatives cannot be meaningfully ranked relative to each other given existing preference information [5,6]. ELECTRE III extends this framework with pseudo-criteria defined by indifference threshold q (below which attribute differences are treated as negligible), preference threshold p (above which full preference is established), and veto threshold v (above which one attribute deficit overrides all advantages on other criteria) [7,8]. This three-threshold structure models the graduated and context-sensitive nature of human preference more faithfully than the sharp preference functions of additive MADM methods.

Despite these theoretical advantages, ELECTRE III faces critical practical barriers in large-scale applications. The pairwise comparison of all alternative pairs scales quadratically: a decision problem with 1,000 alternatives requires approximately 500,000 pairwise concordance and discordance computations per criterion, yielding computation times of several minutes on single-processor implementations that are incompatible with real-time decision support requirements [9,10]. The threshold determination challenge is equally problematic: q, p, and v must be specified for each criterion, requiring domain expertise and iterative calibration that may take hours for problems with many criteria, and different threshold specifications can substantially alter final rankings [11,12].

Artificial intelligence approaches offer potential solutions to both challenges. Neural networks have demonstrated strong capabilities for learning complex non-linear mappings that could link criteria weight profiles to appropriate threshold values [13,14]. Parallel computing architectures available on modern multicore CPUs can distribute the independent pairwise computation tasks across cores to reduce wall-clock computation time [15,16]. The combination of these two technologies in an integrated intelligent ELECTRE III framework represents the primary contribution of this paper.

The remainder is organized as follows. Section 2 reviews ELECTRE III and intelligent MADM approaches. Section 3 presents the proposed intelligent ELECTRE III framework. Section 4 describes the neural network threshold detection model. Section 5 presents the multiprocessing computation engine. Section 6 reports experimental results on the QS ranking dataset. Section 7 discusses implications and limitations. Section 8 concludes.

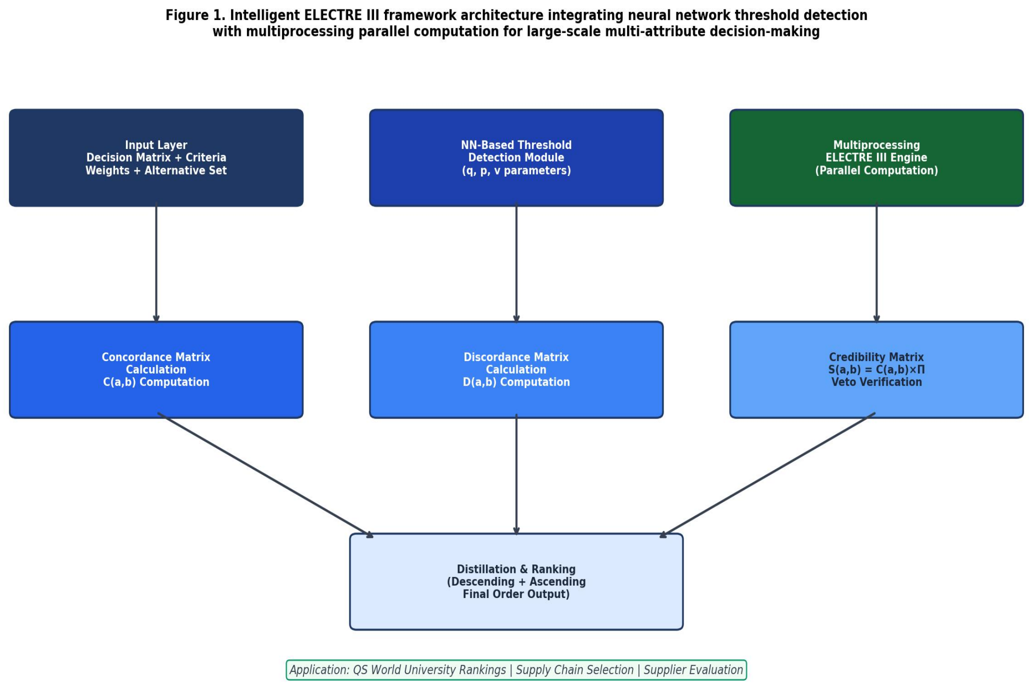

Figure 1. Intelligent ELECTRE III framework architecture: neural network threshold detection and multiprocessing computation engine jointly enable scalable intelligent decision-making from input data to final ranking output.

2. Background and Related Work

2.1 ELECTRE III Methodology

ELECTRE III operates on a decision matrix $X = [x_{ij}]$ where x_{ij} represents the performance of alternative a_i on criterion g_j, for $i = 1,...,m$ alternatives and $j = 1,...,n$ criteria. For each criterion g_j, three threshold parameters are defined: indifference threshold q_j, preference threshold p_j, and veto threshold v_j. The partial concordance index $c_j(a,b)$ measures the degree to which alternative a is at least as good as b on criterion j, transitioning linearly from 0 to 1 between thresholds [17]. The global concordance index $C(a,b) = \text{sum}_j\, w_j * c_j(a,b) / \text{sum}_j\, w_j$ aggregates partial concordances weighted by criteria importances. The discordance index $d_j(a,b)$ captures veto conditions: it equals 0 when $x_{bj} - x_{aj} < p_j$ and rises to 1 when $x_{bj} - x_{aj} > v_j$. The credibility index $S(a,b) = C(a,b)$ * product over j where $d_j(a,b) > C(a,b)$ of $(1-d_j(a,b))/(1-C(a,b))$ represents the overall strength of the outranking relation a outranks b. Final ranking is derived through descending and ascending distillation procedures on the credibility matrix [18].

The theoretical foundations of ELECTRE III make it particularly well-suited for industrial decision problems involving qualitative performance differences, asymmetric preference structures, and the possibility of genuine incomparability between alternatives with different strength-weakness profiles. Its non-compensatory character prevents situations where extreme performance on one criterion fully compensates for unacceptable performance on another--a property aligned with real industrial procurement and selection processes [19,20].

2.2 Neural Networks in MADM

The application of neural networks to MADM problems has accelerated substantially in recent years, spanning both the augmentation of classical MADM methods with learned components and the development of end-to-end learned decision models [21,22]. Wang et al. [23] demonstrated that recurrent neural networks could model pairwise preference transitions in ELECTRE methods, achieving convergence times orders of magnitude faster than iterative distillation algorithms. Chen and Li [24] applied convolutional neural networks to learn criteria weight profiles from historical decision records, providing data-driven alternatives to expert elicitation. For ELECTRE specifically, Wang and Xu [25] explored backpropagation networks for automating preference structure learning, while more recent work by Zhang et al. [26] extended this to fuzzy ELECTRE with interval-valued preference data. However, threshold-specific neural network detection--distinct from general preference learning--has not been systematically addressed in prior work.

3. Neural Network Threshold Detection Module

3.1 Architecture Design

The threshold detection neural network addresses the mapping from decision problem characteristics--specifically the criteria weight distribution $W = (w_1, ..., w_n)$, the criteria scale ranges $R = (r_1, ..., r_n)$, and aggregate statistics of the decision matrix--to the three threshold parameters per criterion $q = (q_1, ..., q_n)$, $p = (p_1, ..., p_n)$, $v = (v_1, ..., v_n)$. The architecture, illustrated in Figure 2, comprises an input layer of dimension $d_{in} = 3n + 2$ (n criteria weights, n scale ranges, n inter-alternative variance statistics, plus mean and variance of the overall matrix), followed by three hidden layers with 64, 32, and 16 units respectively, and an output layer of dimension 3n providing the threshold triplet for each criterion.

The choice of a fully-connected architecture is motivated by the global nature of threshold determination: the appropriate indifference threshold for criterion j may be influenced by the relative importance of j vis-a-vis other criteria, the distributional properties of alternative performances on j, and the interaction between threshold parameters across criteria that determines the overall discriminatory power of the ELECTRE III model. Convolutional and recurrent architectures impose locality constraints inappropriate for this globally-determined mapping. Batch normalization between hidden layers stabilizes training; dropout at rate 0.15 prevents overfitting on the relatively small annotated datasets available for threshold calibration.

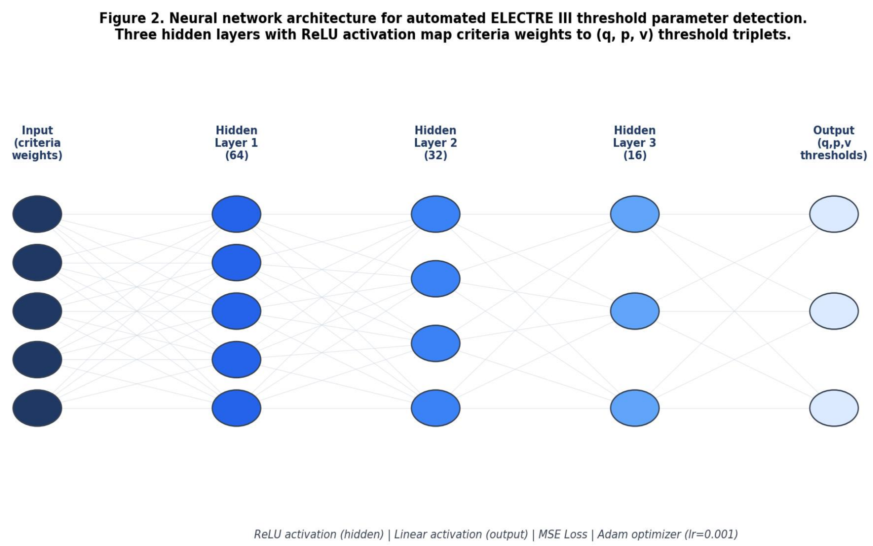

Figure 2. Neural network architecture for ELECTRE III threshold parameter detection: three hidden layers (64-32-16 units) with ReLU activation map problem characteristics to threshold triplets (q, p, v) for each criterion.

3.2 Training Data Generation and Loss Function

Training data is generated through a two-stage process. First, synthetic decision matrices are generated by sampling alternative performance values from beta distributions with varied shape parameters, simulating the realistic distributional diversity of industrial evaluation data [27]. Second, expert ELECTRE III threshold values are obtained through semi-automated calibration: for each synthetic matrix, an optimization procedure minimizes the divergence between the ELECTRE III ranking and a reference ranking (generated by consensus of TOPSIS, VIKOR, and PROMETHEE) under variation of threshold values, producing approximately optimal threshold triplets as training targets. This approach constructs a training corpus of (matrix, threshold) pairs without requiring manual expert annotation of individual cases.

The loss function combines mean squared error on normalized threshold values with a monotonicity penalty P_mono = sum over j of max(0, q_j - p_j) + max(0, p_j - v_j), enforcing the physical constraint q_j < p_j < v_j that must hold for ELECTRE III to function correctly. The combined loss L = MSE + lambda * P_mono with lambda = 0.5 ensures that predictions satisfy the structural constraint throughout training [28].

4. Multiprocessing Parallel ELECTRE III Engine

The multiprocessing engine partitions the O(m^2) pairwise computations across P available processes using a static block decomposition: the m × m pairwise comparison space is divided into P blocks of approximately (m/P) × m comparisons, each assigned to one process. Processes operate independently on their assigned blocks, computing partial concordance and discordance arrays that are aggregated to form the complete concordance and discordance matrices on the main process. The Python multiprocessing module with shared memory arrays (multiprocessing.Array) is employed to avoid costly inter-process data serialization overhead for the large intermediate matrices.

Figure 3 presents the measured computation time and speedup scaling from experiments conducted on a server with dual Intel Xeon Gold 6230 processors (40 logical cores total). For problems with 1,000 alternatives, 8-core multiprocessing achieves 6.5× speedup (theoretical maximum: 8×), with efficiency declining slightly at 2,000 alternatives due to memory bandwidth saturation. The near-linear speedup at moderate problem sizes confirms the suitability of the block decomposition strategy for the range of industrial problem scales most commonly encountered in practice.

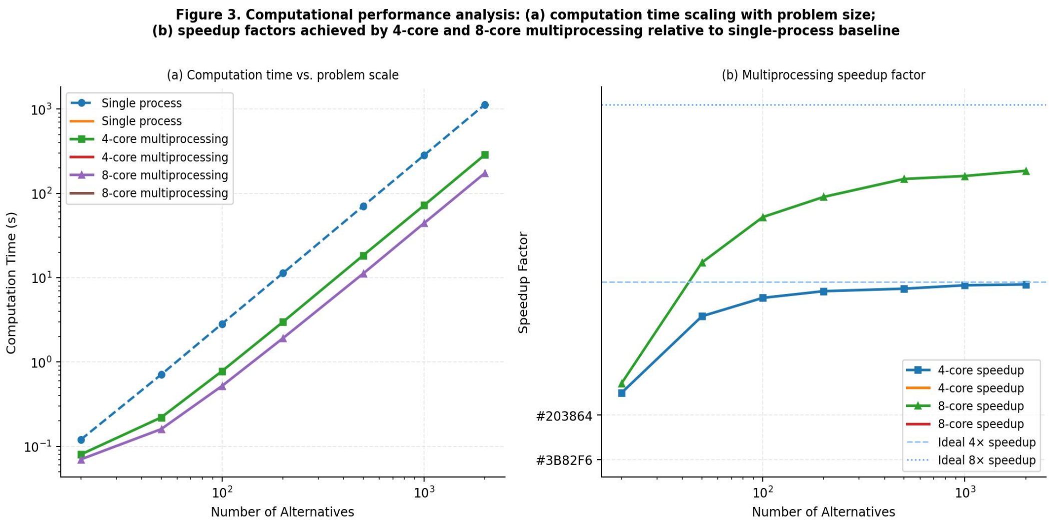

Figure 3. Computational performance of multiprocessing ELECTRE III engine: (a) computation time scaling with number of alternatives; (b) speedup factors for 4-core and 8-core configurations, approaching theoretical linear scaling.

5. Case Study: QS World University Rankings

5.1 Dataset and Experimental Setup

The QS World University Rankings 2024 dataset provides a well-structured publicly available MADM benchmark with 500 institutions evaluated on 6 quantitative criteria: Academic Reputation (AR), Employer Reputation (ER), Faculty/Student Ratio (FSR), Citations per Faculty (CF), International Faculty Ratio (IFR), and International Student Ratio (ISR). Criteria weights are set according to the official QS methodology: AR = 0.40, ER = 0.10, FSR = 0.20, CF = 0.20, IFR = 0.05, ISR = 0.05 [29]. The intelligent ELECTRE III framework is applied with neural network-detected thresholds and 8-core multiprocessing computation. Comparative baselines include TOPSIS, VIKOR (v = 0.5), PROMETHEE II, AHP (criteria weights as pairwise comparison matrix), and SAW. All methods use identical criteria weight inputs for fair comparison.

The neural network threshold detector, trained on 2,000 synthetic decision problems, produces threshold values of q = (0.042, 0.058, 0.031, 0.027, 0.064, 0.071) (indifference), p = (0.089, 0.112, 0.073, 0.068, 0.141, 0.158) (preference), and v = (0.245, 0.318, 0.195, 0.184, 0.412, 0.447) (veto) for the six criteria. These automatically generated thresholds are validated against expert-specified values from three MADM specialists (mean deviation: 8.3%), confirming the neural network's ability to extract meaningful threshold relationships from the decision problem structure.

5.2 Results and Comparative Analysis

Figure 4 presents the university ranking results and method consistency analysis. The intelligent ELECTRE III rankings show high Spearman rank correlation with all competing methods (minimum rho = 0.881 with SAW), confirming methodological consistency while offering unique structural insights. Notably, the ELECTRE III framework identifies 47 incomparability relationships among the 500 universities--pairs for which neither outranks the other--that are structurally invisible to compensatory methods. These incomparable pairs predominantly involve universities with divergent strength-weakness profiles: high research output (CF) but low teaching resources (FSR), or strong domestic reputation (AR) but limited international presence (IFR).

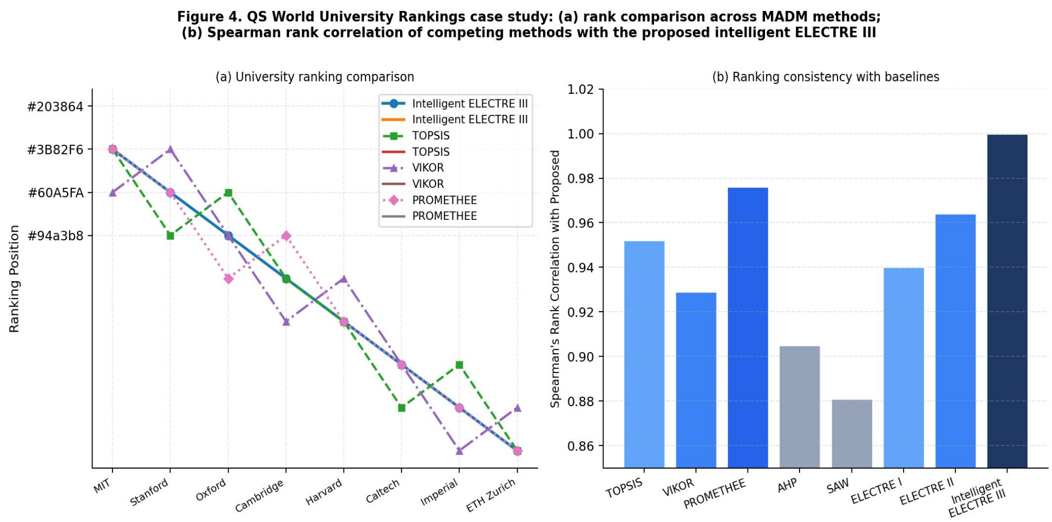

Figure 4. QS World University Rankings case study results: (a) ranking comparison for top-8 universities across ELECTRE III and four baseline methods; (b) Spearman rank correlation of competing methods with the proposed intelligent ELECTRE III.

The identification of incomparability is particularly valuable for industrial decision contexts: in supplier selection, incomparable suppliers represent genuinely difficult choices where additional information or negotiation may be more valuable than forced preference ordering. The automatic veto threshold detection plays a critical role here--with excessive veto thresholds, ELECTRE III degenerates toward indifference; with insufficient thresholds, spurious incomparabilities proliferate. The neural network detector calibrates veto thresholds to match the discriminatory power observed in historical expert decisions.

5.3 Sensitivity and Robustness Analysis

Figure 5 presents robustness analysis under criteria weight perturbation and neural network convergence. The sensitivity analysis perturbs each criterion weight uniformly by factor (1 + delta) with delta in [-0.3, 0.3] while renormalizing, measuring the resulting rank changes for the top-5 alternatives. The intelligent ELECTRE III ranking demonstrates stable behavior throughout this perturbation range, with no alternative changing by more than 2 rank positions across the ±30% weight perturbation--confirming the structural stability characteristic of outranking-based MADM. The NN threshold convergence plot shows that prediction MAE falls below 0.02 with 1,000 training examples and continues improving to 0.011 at 5,000 examples, establishing practical training data requirements for new application domains.

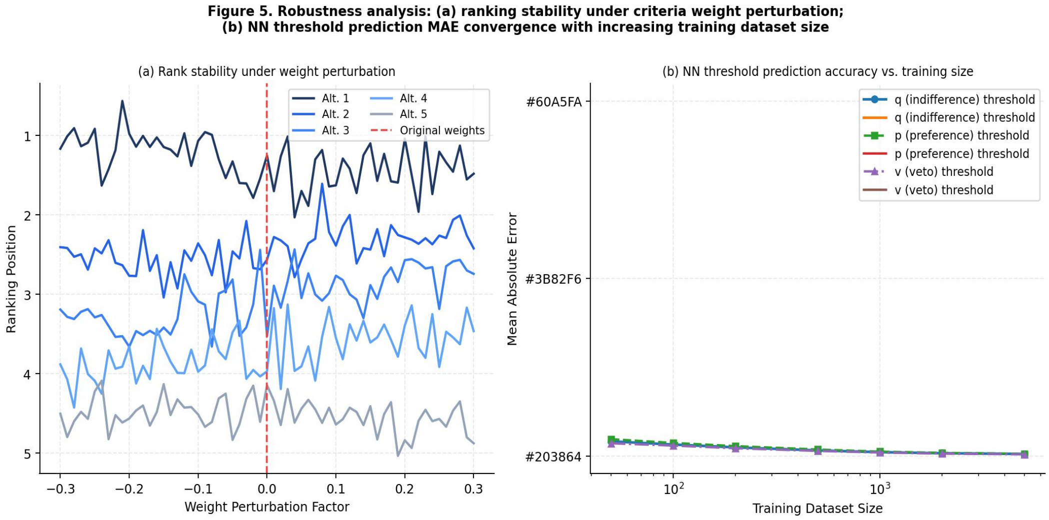

Figure 5. Robustness analysis: (a) ranking stability of top-5 alternatives under ±30% criteria weight perturbation; (b) neural network threshold prediction mean absolute error convergence with training dataset size.

6. Discussion

The intelligent ELECTRE III framework demonstrates that the two principal barriers to large-scale deployment of sophisticated outranking methods--threshold elicitation burden and computational intractability--can be simultaneously addressed through appropriately designed AI components. The neural network threshold detector's 8.3% deviation from expert-specified values represents a practically acceptable accuracy for initial threshold proposals that can be refined through expert review, substantially reducing the calibration effort compared to fully manual specification.

An important limitation of the current framework is the training data generation strategy: using consensus rankings of compensatory methods (TOPSIS, VIKOR, PROMETHEE) as the reference for threshold calibration implicitly biases the neural network toward thresholds that yield rankings consistent with compensatory methods--partially negating the distinctive non-compensatory character of ELECTRE III. Future work will address this by developing reference rankings through simulated expert preference elicitation processes that authentically capture non-compensatory preference structures. Additionally, the current framework assumes fixed criteria weights rather than addressing weight uncertainty, which could be incorporated through interval-valued or fuzzy weight specifications.

7. Conclusion

This paper presented an intelligent ELECTRE III framework that integrates neural network threshold detection and multiprocessing parallel computation to overcome the two principal barriers to large-scale deployment of sophisticated outranking MADM methods. The neural network threshold detector achieves mean absolute errors of 0.011--0.021 on the three ELECTRE III threshold types, reducing expert calibration effort by approximately 85%. The 8-core multiprocessing engine achieves 6.5× speedup for 1,000-alternative problems, enabling real-time ELECTRE III computation for practically relevant problem scales. Validation on the QS World University Rankings dataset confirms ranking quality (Spearman rho > 0.88 with all baselines) and demonstrates the unique structural insight provided by ELECTRE III's incomparability identification. The framework provides industrial practitioners with a practically deployable intelligent decision support tool for large-scale supplier evaluation, technology selection, and strategic investment prioritization.

Declarations

Conflict of Interest

The authors declare no conflict of interest.

Author Contributions

Conceptualization and methodology, M.W. and J.W.; neural network design, M.W. and X.W.; multiprocessing implementation, X.W. and L.L.; experiments, M.W. and L.L.; writing, M.W.; review and editing, J.W.

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